

# An Interstage Change Model for Sandbox Geography

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**Abstract.** Observing children in a sandbox can motivate a new way of designing dynamic spatio-temporal ontologies. Contemporary developmental psychology provides evidence that knowledge about the world is acquired in piecemeal fashion [1,2]. Infants form theory like concepts of the world that are revised in the light of new evidence [3].

These findings can help to build multitiered ontologies grounded in children's spatial experience. Questions of how to structure and connect these ontologies will be addressed. The approach ensures a design that is close to human thinking. In analogy to the human learning process a mechanism to model change between ontologies is proposed.

The formalization of spatial concepts like support and occlusion are investigated in an algebraic framework. In a prototypic implementation a learning agent observes the world – a sandbox. The metaphor of the sandbox stands for a place to carry out experiments and build intermediate models. The development of the agent is described by different stages. A stage can be seen as an interim conceptualization of the world. I hypothesize that each stage can be described by a set of algebras. The change from a stage to another can be expressed by the exchange of axioms within the algebras.

**Keywords:** dynamic ontologies, cognitive development, algebra, computational models

## 1 Introduction

Children's concepts of space are different from adults. Infants are more general in their learning than adults, because their perceptual system is more abstract and less specific [4]. It can be assumed that children start out with a certain knowledge into this world and that there are mechanisms of development that make a human proceed through life.

Imagine a world in which objects magically disappear and reappear! It's a world we all have been living in when we started being around. Developmental psychology made big progress in the last decades in investigating infants conceptualizations of the world. An uncountable amount of empirical experiments

has been carried out to explain the behavior of children and give an account for cognitive development.

In the search for a naive geography with formal models that are closer to human thinking [5,6] these accounts should be considered. All adults have once been children and have been going through development processes. The findings of developmental psychology should be incorporated in ontologies for geoinformation systems. Two directions of research are motivated by their results. First to study the structure of spaces, built up by operations and perceptions of children in small scale space. This includes the formal description of such spaces. Second to connect the different conceptualizations to a learning theory for a cognizing agent, e.g., based on a re-weighting mechanism as suggested in the literature [7].

This paper will concentrate on the first part of the question, namely how to structure the ontologies, in order to build a proper base for investigating the change mechanism between ontologies. Stagewise acquisition of spatial knowledge has been already proposed very early [8,9] e.g., children go through six stages when learning about the permanence of objects. Some of these findings could be worked out in greater detail since the early findings of Piaget under the aspect of large scale space [10,11]. In a recent view conceptualizations of the world are explained as theories [12,3]. These theories change in the light of new evidence. To distinguish them from fully fledged theories, philosophers like Roberto Casati would rather call them “theoritas” (little theories) [13].

For the present paper a theory stands for a conceptualization of the world at a certain timepoint in an infants development. In early infants behavior spatial relations seem to be missing. Concepts like containment, support or occlusion are gradually learned by making sensorimotor experiences in space. Examples for such small theories are worked out within the paper and their structure is investigated.

The aim of building ontologies that are based on children’s concepts of space is a contribution to the area of naive geography and spatial reasoning. In the present example a static agent (does not move) observes the world. At first sight it seems hard to explain that problems in the “real world” can be conceptualized using this paradigm. But it is one explanation how children start out in the world, building concepts about objects and persons. Small theories that develop to complex theories.

Complex systems can be build from simple parts, this applies also for geoinformation systems [14]. The simple parts have been identified in a series of studies about humans conceptualization of space. The crucial question however is how these parts are linked and interact with each other. By searching a learning theory we hope to find mechanisms that describe the structure of and the connections between these parts.

The remainder of the paper is organized as following: Section two reviews related work in conceptualization of space and models proposed for it. Section three proposes an overall framework for an object ontology that is modular, hierarchical, dynamic and action-driven. The connection to algebra is discussed.

Section four gives deeper insights in the modules and the modelling process. A computational model is presented in section five. Section six concludes the paper and gives an outlook to future research questions.

## 2 Human Conceptions of Space

This section reviews what is known about the human conceptualization of space and how far it has been considered in formal models. The existence and structure of space has been of continuous research interest in geoinformation science. Conceptualizations of space have been worked out [15,16,17,18,19,20], mainly distinguishing between small and large scale space resulting in various taxonomies of spaces.

Freundschuh and Egenhofer study the links between different spaces. They base their study on a review of 15 different models of (geographic) space. The attributes: size of space, manipulability, and locomotion are used to propose a model that distinguishes six types of spaces and the connection between them. One of the open research questions that has been left is to explore what kind of spatial knowledge people acquire in each kind of space [19].

Couclelis and Gale discuss the formal difference between perceptual and cognitive spaces. Utilizing an algebraic approach they point out the difficulty to find a universal definition for the concept of space. Different spaces have different algebraic structures. Physical space is a group while e.g., sensorimotor space is a monoid [17].

The various definitions of space seem to appear as humans are multimodal in their location coding. Four modes of location coding (see table1) have been suggested, acting in a hierarchical framework, at the same time at different levels of spatial resolution and accuracy [7]:

- *Egocentric learning* is also called sensorimotor learning or response learning. It comprises the association of a goal with a particular pattern of muscular movement. The pattern is executed like a script on a computer, ignoring outer influence like landmarks or external knowledge. A well-known route is encoded simply by series of linear movements and turns.
- *Dead Reckoning* or inertial navigation stands for the ability of a human and various other organisms to store a self-referenced position with with a number of distance/direction movement pairs.
- *Place Learning* is defined as location coding with external landmarks in connection with distance and/or direction information.
- *Cue Learning* is the storing of object locations with associations. E.g., keys or documents have a habitual place in a home. The location is not necessarily a point location. It can also be an area, due to the hierarchical location coding mechanism humans apply.

The present work concentrates on small scale space and the various conceptions in a child's development of it, hereby simple methods (table 1) will be investigated. Recent formal models of tabletop space have been based on image

|                   | Self-referenced       | Externally referenced |
|-------------------|-----------------------|-----------------------|
| Simple, limited   | sensorimotor learning | cue learning          |
| Complex, powerful | dead reckoning        | place learning        |

**Table 1.** Location coding in infants taken from Newcombe and Huttenlocher [7].

schemata [21,22] in toy spaces. The proposed model differs in so far as it considers infants activities in space. Empirical experiments carried out in developmental psychology, serve to model ontologies for objects in a sandbox.

The formalization of cognitive development has been exploited as a tool for psychologists to build sound theories. A sound theory is the result of very specific definitions, as used in computer models. Developmental psychology also inspired the research in autonomous agents and robots. Models using production systems [23,24] concentrated on describing the stages of development, without explaining the shift between different levels of competence. More recent studies investigate qualitative changes in development using connectionist approaches like neural networks [25,26,27,28].

There is a multitude of concepts for space and spatial objects. It seems that humans are endowed with various mechanisms to code them. The learning process of infants can be a motivation to formalize object ontologies that are close to human thinking. Different levels of competence can be identified in infants development. A sound mechanism to describe the transition between these levels is still an open research question. A transition mechanism would enable to build a framework that allows to switch between different levels of competence and between different kind of spaces. Computational models can help to define sound theories and provide feedback for necessary investigations.

### 3 Structure of the Ontology

This section describes an object ontology for tabletop space. It is built by different modules, that represent the knowledge of an agent. The modules are hierarchical, referring to different rulesets. Changes in the conceptual model of the world are reflected through changes in the rulesets, thus making the ontology dynamic. The ontology is also action driven, as the agent acts in the world and gains knowledge through experience.

#### 3.1 A modular agent that observes the world

An agent can build theories about the world through observation. The current model assumes that this process is free of any error or uncertainty. Each theory is defined by an initial concept. The world is represented by a sandbox. It is a simplified toy world that allows to model magic objects. These objects do not fall or have no friction while moving. Events define interactions between objects at certain timepoints. Events comprise spatial relations like occlusion or

containment. Each observation of an event in the world can give evidence for or against the concepts of the agent. In order to model beliefs of the agent and facts in the world, a multilayered ontology is applied [29].

The agent carries out an “observe-predict-check” function cycle. A predict function transforms the agents’ perceptions of objects into expectations of events in the sandbox. The check function helps the agent to verify its internal status. Expectations of events (object A falls) are compared with actual facts (object A does not fall, it is still supported). The agent is able to internally grade the concepts and elicit changes through a growUp function.

The agents’ knowledge is build from modules (see figure 1). In principle all modules are available from the start. Each module stands for a theory. There is a distinction between core theories and add-on theories. Core theories describe basic object behavior like movement and identity. More advanced concepts like occlusion, containment, friction, gravity, and weight are described in add-on theories. All concepts underlie change apply the “observe-predict-check” mechanism described.

**Example 1. Algebra agent where**

```
observe :: agent->Sandbox->Time->agent
predict :: agent -> agent
check :: agent-> agent
growUp:: agent -> agent
```

### 3.2 The structure of the agents knowledge

Developmental psychology gives evidence that this knowledge is acquired in piecemeal fashion. An agent defines theories. The theories are modules and refined in a coarse-to-fine strategy as the agent acts in the environment. Early theories have little information elements and few links between modules.

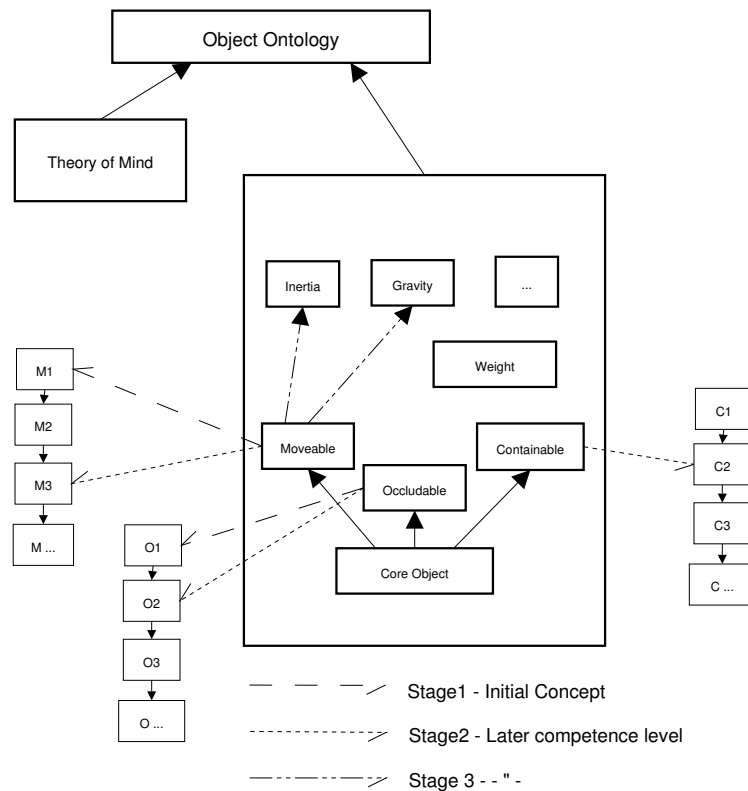
As time proceeds more and more details are added, some information is stored redundantly. An agent raises his level of competence by adding new constraints to existing concepts. At a certain time point an agent is endowed with differently developed theories. Some of the theories are still in an initial state, some are more advanced. A seven months old infant may understand object support but not object containment. The knowledge at a certain time point will be called *stage*. Jean Paul Piaget defined development, based on children’s behavior in stages [30].

Production systems provide a computational model for stagewise development [23,24]. They are based on if-then-else rules and rather describe the behavior of an infant than mechanisms for development.

Algebraic specifications can be an alternative. Algebras are defined as a set of sorts, operations, and axioms [31]. The functional programming paradigm with algebraic specifications focuses on operations, thus activities carried out in space. The approach provides an object oriented view to the world, where algebras group operations based on the same data type. The advantage of using

algebra is its mathematical sound- and compactness. It allows the reuse of code by defining sub algebras and combining different algebras [14].

Each theory about the world is described with algebraic specifications. A change in the conceptualizations of the world is reflected in an exchange of axioms. Examples of spatial relations are given in section four. Further models should bring up general axioms that allow to define structure preserving mappings between the different algebras. Such mappings are called morphisms and would mathematically explain connections between different concepts.



**Fig. 1.** Structure of the ontology

### 3.3 A dynamic ontology

Each stage of the agents' knowledge comprises a number of interlinked modules. Like in a neural model based on modules the number of links grows as the knowledge of the agent does (see figure 1).

Knowledge at a certain stage is described by the axioms of the different algebras. A weighting mechanism based on probability will be investigated to elicit the exchange of axioms. Observations lead to predictions and measures for the appropriateness of certain axioms. A stochastic approach like Bayesian Belief Networks may explain the development between the different stages [32]. The change mechanism in the cognitive model of the agent is a topic of future research.

### 3.4 Actions in the world

An agent defines theories while acting in the world. Activity is an important element of development [33,34]. At the moment the only action considered in the model is observation, in simplified error free form. The present model can benefit in future from the consideration of mobility and other perceptions towards a sensorimotor model of an agent. Considering the activities and the intention of a user leads to “action-driven ontologies” [35,36]. A specific set of theories may be sufficient to solve a given problem.

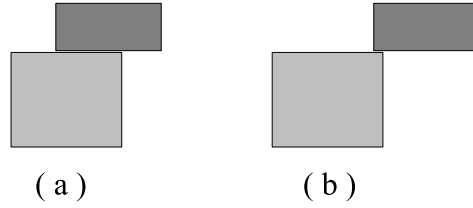
## 4 From Rulesets to Algebra

One of today's views of cognitive development is that children possess innate knowledge that is triggered by the environment and by powerful learning capabilities<sup>1</sup>. Researchers interpret the reaching and looking behavior of children. Using this paradigm it is assumed that children look longer at unknown or new events than to experimental situations they are familiar with. In order to test certain knowledge, children are confronted with manipulated physically possible and impossible events (see figure 2) [37,12,38,39]. Their behavior is interpreted as knowledge.

An *incremental knowledge account* has been proposed, suggesting that knowledge is event specific. Events are interactions between objects in space e.g. support, occlusion and containment [38]. Infants seem to apply a coarse-to-fine strategy for each of these events and develop knowledge in terms of variables. Variables are, e.g., the height and the width of an object or the amount of contact between two objects. Children that seem to be aware of variables in one event, do not necessarily use them in other events. Four months old children seem to infer about the occlusion of objects via width and height, but do not do so in a containment situation [40]. Renee Baillargeon tested the impossible event and also inverted the test situation. In an incremental knowledge account some possible

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<sup>1</sup> Among developmental psychologists there are discussions to what degree knowledge is available in advance and how much do learning processes and environment contribute to development. The extreme standpoints of nativism (all knowledge is available in advance) and empiricism (no knowledge is available, it is acquired through the environment) is hardly taken by any researcher. Contemporary accounts are in between these standpoints. This is also the authors current view of cognitive development.



**Fig. 2.** Support with (a) possible and (b) impossible event. Children having learned about the amount of contact on top of a supporting object, should expect that the dark grey object falls down in event (b). Therefore they will look longer at the situation in event (b) than in event (a).

situations are expected impossible, due to missing knowledge [2]. This could be verified by experiments. These results and the hierarchical framework of location coding, reviewed in section two strengthens the view that a conceptualization of the world can be build on small theories.

#### 4.1 Core objects

The basic elements of the object ontology are core objects. They consider the identification and the comparison of objects. There is some evidence that newborns distinguish objects based on their spatio-temporal properties [4,3,2]. Empirical tests show that infants at the age of a few months can identify objects via spatio-temporal attributes and move later on to a feature based object identification. This has been expressed by exchangeable rulesets [41,4]:

- Rule1: An object is a bounded volume of space in a particular place or on a particular path of movement.
- Rule 2: An object is a bounded volume of space of a certain size, shape, and color that can move from place to place along trajectories.

These rules are translated in algebraic definitions. An example for an algebra of moveable objects is given (example 2). The algebra is very simple and has just one axiom.

*Example 2. Algebra Moveable o where*

$isAt::o \rightarrow t \rightarrow pos$

$speed::t \rightarrow o \rightarrow v$

**Axiom:**

$isAt\ t2\ o1 = isAt\ t1\ o1 + (t2 - t1)\ speed\ (t1\ o1)$

Operations to set and get the size, shape, and color of an object are added. The extended algebra can be seen below.



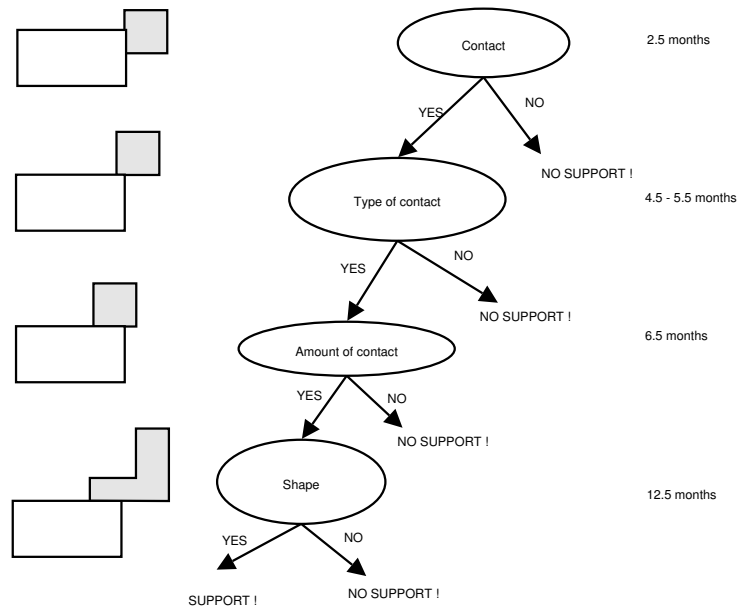
**Example 3. Algebra Moveable  $o \Rightarrow$  Algebra Properties  $o$  where**

```

makeObj :: ID -> Color -> Size -> Shape -> Pos -> o
setColor :: o -> Color -> o
setSize  :: o -> Size  -> o
setShape :: o -> Shape -> o
getColor :: o -> Color
getSize  :: o -> Size
getShape :: o -> Shape
    
```

**4.2 Supportable objects**

Knowledge about the support of objects has been tested in interpreting children’s looking time at novel events. An initial concept for support seems to consider the contact between two objects. Figure 3 shows a decision tree for the support of two objects. Each node represents a variable (see section four). Traversing deeper into the decision tree reduces impossible expectations. The left side of the figure shows impossible outcomes. The right side of the figure indicates the ages. The grey object supported in figure 3 is not expected to fall, although the objects do not contact on top of the supporting surface (3 months), the amount of contact is to small (6.5 months) and the shape does not allow support (12.5 months) [38].



**Fig. 3.** Decision tree for the support of objects [38].

The axiom in the example below considers a contact of two objects at the top surface.

*Example 4. Algebra Support c where*

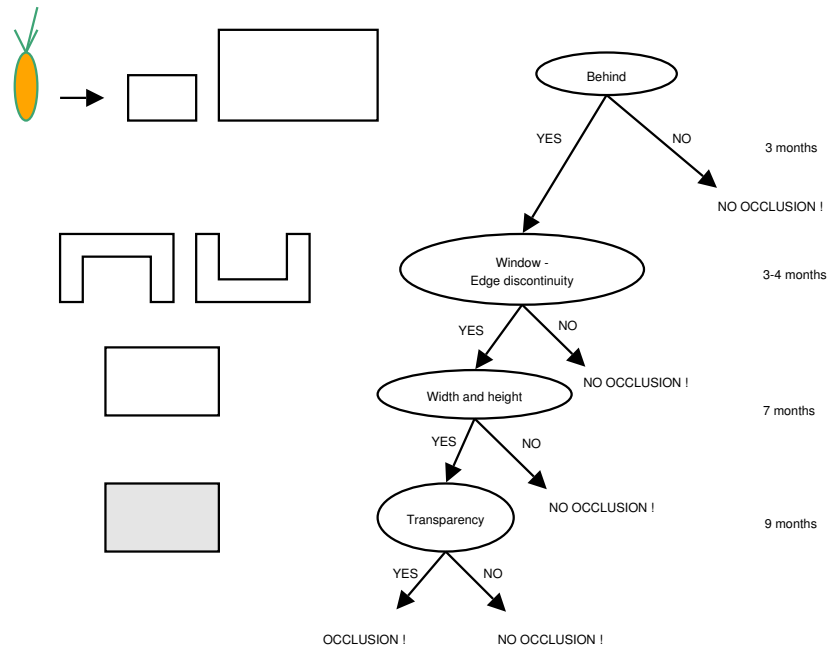
```
makeContact::Obj->Obj->Con_Type->Amount->Cg->c
  getConObjs::c->(Obj,Obj)
  getConType::c->Con_Type
  getConAmount::c->Amount
  getConCg::c->Cg
  isOn::c->Bool
Axiom:
  isOn = getConType c == Top
```

### 4.3 Occludeable objects

Occlusion events are defined as events in which one object moves or is placed behind a nearer object (occluder). The same method of research as with supportable objects has been applied. Initially infants (2.5 months old) seem to expect an object as occluded when it is behind a closer object. At the age of three months children seem not to cognize all types of rectangular holes in occluders. Four months old infants start to use width and height attributes of objects in occlusion events. At the age of 7.5 months transparency is recognized as a salient variable [38].

Figure 4 shows a decision tree for the occlusion of objects. Each node represents a variable (see section four). Traversing deeper into the decision tree reduces impossible expectations. The left side of the figure shows impossible outcomes. The right side of the figure indicates the ages.

The orange object moving behind the screen in figure 4 is not expected to appear, although the occluder has a window (upper - lower edge), the moving object is bigger than the occluder, and in the last case the occluder is transparent.



**Fig. 4.** Decision tree for the occlusion of objects [38].

An advanced theory of occlusion of two objects distinguishes objects in front and behind, windows in the occluder, the amount of occlusion, width and height of the occluder, as well as the transparency of the occluder. The axiom in the example below defines occlusion where an object is located behind another object.

*Example 5. Algebra Occlusion o* where

makeOcclusion::Obj->Obj->Occ\_Type->Amount->Occ\_Width->Occ\_Height->Property->o

getOccObjs::o->(Obj,Obj)

getOccType::o->Occ\_Type

getOccAmount::o->Amount

getOccWidth::o->Occ\_Width

getOccHeight::o->Occ\_Height

getOccProperty::o->Occ\_Property

isHidden::o->Bool

**Axiom:**

isHidden = getOccType o == isBehind

#### 4.4 Other Objects

Other rulesets have been translated to algebraic specifications. Containment has been investigated by lowering an object into a container. Initial rules about containment check of the top surface of the container is open and whether the object is lowered behind or in the container [40,2].

Infants' theories of object motion under the influence of inertia and gravity have been studied and rules can be defined. An object that was arrested and has been released has to move down (fall), as well as it has to accelerate appropriately. Objects that roll down a cliff have to move on a parabolic trajectory [42,1].

Children have been given balancing tasks with a fulcrum. The development process could be described by a set of four rules. The first rule just considered weight, the second rule additionally distance. In a third rule conflict between weight and distance was handled by guess. The fourth rule could predict the torque such that the correct answers could be given [43].

The development of *counting objects* is described in a stagewise process. Infants start with symbolic concepts, like "one", "two", "many" and move in a later stage to order relations. The final stage uses a rule like successor = predecessor + 1. [44]. The analogy to the algebra of natural numbers is obvious without further explanations.

The various experiments show that infants' knowledge about objects can be defined in rulesets. A translation of the rulesets to algebraic specifications leads to sound mathematical models that can be executed using the functional programming paradigm. Further models will be built to extend the object ontology.

### 5 Computational Model

A computational model of an agent in a "sandbox" has been realized. It serves as a proof of concept for the proposed theory. The functional programming paradigm was chosen, that allows rapid prototyping with algebraic specifications. A static agent perceives snapshots of a tabletop world and carries out reasoning based on the conceptual model described in section three. A finite state machine presents the current model of the agent.

**Definition 1.** *data Agent = C\_Agent AgentID AgentState [Percept] [Fact]*

Each theory of the world is represented as a class. Yet the case of contradicting axioms in the theory has not been discussed. The lazy evaluation mechanism in functional programming languages allows a termination of the program whenever one of the expressions terminates. A solution is also guaranteed in the case of contradicting axioms. The implications of conflicting situations did not occur in the modeling process yet and has to be studied in the future.

**Definition 2.** *Different instances of the Agent implement the behavior at different stages.*

```

instance Stage1 Agent where
  isEqual1 ag o1 o2 = (getPos o1 == getPos o2)
  isOn1 ag con = (getConType con == Top) || (getConType con == Side)
  ...
instance Stage4 Agent where
  isEqual4 ag o1 o2 = (getPos o1 == getPos o2) &&&(getAttrib o1 == getAttrib
o2) &&& ...
  isOn4 ag c = (getConType c == Top) &&& (getConAmount c > 35) &&&
(getConCg c == Cg_On)

```

A module for test data allows to define different setups. Experiments carried out in developmental psychology can be defined and the inferred interpretations validated. The implementation of further experiments will refine the computational model.

## 6 Conclusions

The present model is able to simulate a spatial cognizing agent in a sandbox. Theories of developmental psychology can be tested in a rapid prototyping manner. Simple parts have been identified that contribute to an object ontology that is close to human thinking. Equality, occlusion, and support of objects have been defined and formally investigated. Like physicists searching for the smallest parts of the universe, geoscience is on the way to identify the smallest elements of processes in space. Using the results of developmental psychology I proposed the design of an ontology having four properties:

1. modular: A conceptualization of the world is build from theories. Each theory is presented in a cognitive model as a module that can change. Initial concepts are available to describe the world.
2. hierarchical: Theories of the world can be mapped in hierarchical tree like structures. Additive Information can be inserted into the model, to raise the level of competence of the agent.
3. dynamic: Theories are expressed by algebras. A mechanism of axiom exchange allows to change the behavior of the algebras, thus the spatial conceptualizations of the world.
4. action driven: The acting of the agent in the environment elicits changes in the model, by giving the agent evidence for his concepts.

On the way of defining small theories about the world, children undergo changes. Children have a early notion of gravity (things fall down), when children learn to sit they also seem to get an understanding for support relations. The importance of mobility in development has been studied [33,34]. This would provide a link to large scale spaces in the proposed theory. Another aspect is the acquisition of language, that has influences on the conceptualization of the world, e.g., the development of counting [44]. Extensions in one of these directions are a potential topic for future research.

People have different beliefs about the world, this could be modeled in an multiagent system. Agents have also beliefs about the beliefs of other agents. Psychologists call this the theory of mind. A formal model of the theory of mind is necessary to complement an object model in an information system that is close to human thinking.

The characteristics of thought processes have to be further investigated and formal models of learning theories developed. Candidates for formalisms are fuzzy logic [45] or many valued logic. Alison Gopnik proposed a mechanism based on Bayesian Belief Networks [32]. In future work (hidden) Markov Models [46] will be investigated as a transition mechanism between the different stages.

The structure of the spatial environment is hierarchical, but not necessarily tree like [47,48]. Research in how to structure the spatial environment, degree of spatial resolution, etc. to appropriately map the cognitive model with the real world is necessary.

I am confident that there are several possibilities to link the ontologies and that the mechanism carries a certain redundancy. The consistent mathematical framework of algebra should be further exploited.

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